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Study on the inertia effect of helical spring of the absorber on suppressing the dynamic responses of a beam subjected to a moving load

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Abstract

In the dynamic analysis of the *classical absorber* composed of a spring, a damper and a lumped mass, the mass of its helical spring (or spring mass) is usually neglected. The purpose of this paper is to present a *modified absorber* with inertia effect of the spring mass considered. In order to evaluate the vibration–reduction efficiency of the presented modified absorber, the forced vibration analyses of a single degree-of-freedom (dof) and a multiple dof structural systems are performed. The first analysis is to determine the dynamic magnification factor of a single dof spring–mass system, respectively, attached by a classical absorber and a modified absorber and subjected to a harmonic excitation. The second analysis is to determine the maximum dynamic responses of a multiple dof pinned–pinned beam, respectively, attached by the last two kinds of absorbers and subjected to a moving concentrated load. Because the second analysis is conducted using the conventional finite element method (FEM), in additional to the mass matrix, damping matrix and stiffness matrix of the classical and modified absorbers, the expressions for calculating the optimum parameters of the last two absorbers associated with any order of vibration mode of the pinned–pinned beam are also derived based on the modal data obtained from the mode superposition methodology and the orthogonal property between the normal mode shapes. Numerical results reveal that the spring mass of the absorber has the effect of suppressing the maximum dynamic responses of the main structural system and should be considered in the formulation to agree with the practical situation.

1. Introduction

Because tuned mass damper is a widely used passive vibration absorber for suppressing the dynamic responses of a structure due to external dynamic loads, many researchers have studied the relating problems. For example, Ormondroyd and Hartog [1], Brock and Mo [2], Hartog [3] and Warburton [4] have derived the optimum parameters of the absorber for suppressing the dynamic responses of the single degree-of-freedom (dof) spring–mass main system. In these researches, the absorber was considered as a spring–damper–mass system and directly attached to the main system. Ren [5] has developed an alternative method for the vibration

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reduction of a single-dof main system due to external loads. In this research, the damper of the absorber was connected to the ground (or the base structure) rather than the main mass system. It is well known that, any real structure can be reasonably represented as a multiple dof system, but if the contributions of higher modes to its dynamic responses are negligible, then the last multiple dof system can be replaced by an equivalent single dof main system. In such a case, the optimum absorber parameters can be determined based on the vibration–reduction efficiency of the equivalent single dof main system due to external loads. Based on the last concept, Yau and Yang [6], Joshi and Jangid [7], Rana and Soong [8], Rice [9] and Kwon et al. [10] have used the above-mentioned optimum absorber parameters for the equivalent single dof main system [1–4] to suppress the dynamic responses of the multiple dof structures subjected to earthquake loadings and moving loads. From the review of the preceding literature, it is found that the helical spring of the absorber is assumed to be massless so that the mass of its helical spring (or spring mass) is neglected in the analysis. To improve this drawback, the inertia effect of the spring mass of the absorber is considered in this paper.

First of all, by considering the spring mass of the absorber, the equations of motion of a single-dof spring–mass main system and the attached spring–damper–mass absorber, are derived using the Lagrange's equations. Then, the optimum parameters for vibration reduction of the single-dof main system due to an external harmonic load are determined. Next, based on the modal data obtained from the mode superposition methodology and the orthogonal property between the normal mode shapes of the multiple dof pinned–pinned beam, the technique for determining the optimum absorber parameters associated with any order of vibration mode of the beam are presented. Finally, the last optimum absorber parameters are used for suppressing the dynamic responses of a pinned–pinned beam subjected to a moving load. Numerical results show that the inertia effect of the spring mass of the absorber does affect the vibration–reduction efficiency of the absorber to some degree. Because the helical spring of the absorber is not massless in practice, the optimum parameters of the modified absorber with inertia effect of the spring mass considered should be more reasonable than the ones of the classical absorber with inertia effect of the spring mass neglected.

For convenience, in this paper, an absorber is called the *modified absorber* if its spring mass is considered and is called the *classical absorber* if its helical spring is assumed to be massless. Besides, a beam is called the *loaded beam* if it carries any number of absorbers and is called the *bare beam* if it carries nothing.

2. Equations of motion for the main system and the attached modified absorber

Fig. 1 shows a spring–damper–mass absorber attached to a single dof spring–mass main system. In which, m_1 and k_1 are the mass and spring constant of the main system, respectively, m_2, c_2 and k_2 are the mass, damping coefficient and spring constant of the absorber, respectively, $\bar{m}_s$ and ℓ_s are mass per unit length and total length for the helical spring of the absorber, respectively, x_1, $\dot{x}_1$ and $\ddot{x}_1$ are the displacement, velocity and acceleration of the main system, respectively, while x_2, $\dot{x}_2$ and $\ddot{x}_2$ are the corresponding ones of the absorber. Because the object of this paper is to determine the optimum parameters of the absorber, only the inertia effect of the spring mass of the absorber is considered in this section.

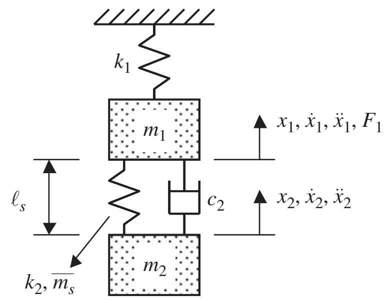

Fig. 1. A spring–damper–mass absorber (k_2, c_2 and m_2) attached to a single degree-of-freedom spring–mass main system (k_1 and m_1).

The kinetic energy T and potential energy V of the entire vibrating system, as shown in Fig. 1, are given by

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} \int_0^{\ell_s} \bar{m}_s \left[\dot{x}_1 + \frac{x}{\ell_s} (\dot{x}_2 - \dot{x}_1) \right]^2 \mathrm{d}x, \tag{1}$$

$$V = \tfrac{1}{2} k_1 x_1^2 + \tfrac{1}{2} k_2 (x_2 - x_1)^2. \tag{2}$$

Note that the displacement of the infinitesimal spring element dx for the helical spring of the absorber is assumed to linearly vary from x_1 to x_2 in the vertical direction. Besides, the total length of the helical spring of the absorber, ℓ_s, refers to that with respect to the static equilibrium position. In the conventional finite element method (FEM), the property matrices of a spring element are similar to those of a longitudinally (or axially) vibrating pin-jointed bar element [13]. Thus, the above-mentioned assumption for Eq. (1) is the same as that for the pin-jointed bar element in FEM [13].

Substituting Eqs. (1) and (2) into the following Lagrange's equations [11]:

$$\frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{x}_1} \right) - \frac{\partial T}{\partial x_1} + \frac{\partial V}{\partial x_1} = c_2 (\dot{x}_2 - \dot{x}_1) + F_1, \tag{3a}$$

$$\frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{x}_2} \right) - \frac{\partial T}{\partial x_2} + \frac{\partial V}{\partial x_2} = c_2 (\dot{x}_1 - \dot{x}_2) \tag{3b}$$

one obtains

$$m_1 \ddot{x}_1 + \frac{m_s}{6} (2\ddot{x}_1 + \ddot{x}_2) + c_2 (\dot{x}_1 - \dot{x}_2) + k_1 x_1 + k_2 (x_1 - x_2) = F_1, \tag{4a}$$

$$m_2 \ddot{x}_2 + \frac{m_s}{6} (\ddot{x}_1 + 2\ddot{x}_2) + c_2 (\dot{x}_2 - \dot{x}_1) + k_2 (x_2 - x_1) = 0, \tag{4b}$$

where F_1 is the external force applied on the main system and m_s represents the total mass of the spring, i.e.,

$$m_s = \bar{m}_s \ell_s. \tag{5}$$

Eqs. (4a) and (4b) are the equations of motion for the main system and the attached (spring–damper–mass) absorber.

Setting $m_1 = k_1 = 0$ in Eqs. (4a) and (4b), and writing the resulting equations in matrix form, one has

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{f\}, \tag{6}$$

where

$$\{\ddot{x}\} = [\, \ddot{x}_1 \quad \ddot{x}_2 \,]^{\mathrm{T}}, \tag{7a}$$

$$\{\dot{x}\} = [\, \dot{x}_1 \quad \dot{x}_2 \,]^{\mathrm{T}}, \tag{7b}$$

$$\{x\} = [\, x_1 \quad x_2 \,]^{\mathrm{T}}, \tag{7c}$$

$$\{f\} = [\, F_1 \quad 0 \,]^{\mathrm{T}}, \tag{7d}$$

$$[m] = \begin{bmatrix} \frac{m_s}{3} & \frac{m_s}{6} \\ \frac{m_s}{6} & m_2 + \frac{m_s}{3} \end{bmatrix}, \tag{7e}$$

$$[c] = \begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, \tag{7f}$$

$$[k] = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}. \tag{7g}$$

In the last formulations, Eq. (6) is the equation of motion of the modified absorber, while [m], [c] and [k] are, respectively, the mass, damping and stiffness matrices of the modified absorber with inertia effect of its spring mass considered. Note that setting $m_1 = k_1 = 0$ is only for the convenience of derivation of the mass matrix of the modified absorber, [m]. From the element mass matrix of the absorber, [m], given by Eq. (7e) one sees that one third of the spring mass of the absorber, $\frac{1}{3}m_s$, will affect the lumped mass of the absorber, m_2, and the other $\frac{1}{3}m_s$ will affect the mass of the spring–mass main system, m_1. Besides, $\frac{1}{6}m_s$ has also the effect of coupling the spring–mass main system and the modified absorber. Because the spring mass of the modified absorber, $m_s = \bar{m}_s \ell_s$, is the *distributed* mass and the lumped mass of the absorber, m_2, is the *concentrated* mass, it is evident that the effect of using a slightly higher value of m_2 itself is different from separately defining m_s.

From Eqs. (7e)–(7g) one sees that the property matrices of the absorber element can also be derived from the coupled equations defined by Eqs. (1) and (2). Thus, the current section provides an alternative approach for deriving the property matrices of an absorber element in addition to the conventional approach used in the FEM [13].

3. Dynamic magnification factor and optimum parameters for the modified absorber

In this section, the mathematical expression for the dynamic magnification factor of the lumped mass m_1 of the main system, $|\bar{x}_1/x_{st}|$, is derived first. Then, the optimum frequency ratio f_{opt} and the optimum damping ratio ξ_{opt} are determined, in turn, from the last expression.

3.1. Dynamic magnification factor for the lumped mass m_1 of the main system

If the lumped mass m_1 of the main system is subjected to a harmonic force

$$F_1(t) = \bar{F}_1 e^{j\omega t}, \tag{8a}$$

then the steady-state responses of the main system and the absorber, $x_1(t)$ and $x_2(t)$, take the form

$$x_1(t) = \bar{x}_1 e^{j\omega t}, \tag{8b}$$

$$x_2(t) = \bar{x}_2 e^{j\omega t}. \tag{8c}$$

In Eq. (8), $\bar{F}_1$ and ω represent the amplitude and forcing frequency of the external force $F_1(t)$, respectively, while $\bar{x}_1$ and $\bar{x}_2$ represent the amplitudes of $x_1(t)$ and $x_2(t)$, respectively.

Introducing Eqs. (8b) and (8c) into Eqs. (4a) and (4b), one has

$$(-m_1^*\omega^2 + k_1 + k_2 + j\omega c_2)\bar{x}_1 - \left(\frac{m_s}{6}\,\omega^2 + j\omega c_2 + k_2\right)\bar{x}_2 = \bar{F}_1, \tag{9}$$

$$-\left(k_2 + \frac{m_s}{6}\,\omega^2 + j\omega c_2\right)\bar{x}_1 + (j\omega c_2 + k_2 - m_2^*\omega^2)\bar{x}_2 = 0, \tag{10}$$

where

$$m_1^* = m_1 + \frac{m_s}{3}, \tag{11}$$

$$m_2^* = m_2 + \frac{m_s}{3} \tag{12}$$

are the *effective lumped masses* of the main system and the modified absorber, respectively. Because the goal of this research is to minimize the responses of the main system, only the response amplitude $\bar{x}_1$ is interested herein. Solving Eqs. (9) and (10) for $\bar{x}_1$, one obtains

$$\bar{x}_1 = \frac{K_2 + j\omega c_2}{[K_1K_2 - (m_2^* + \frac{1}{3}m_s)k_2\omega^2 - \frac{1}{36}m_s^2\omega^4] + j\omega c_2[k_1 - (m_2^* + \frac{1}{3}m_s)\omega^2 - m_1^*\omega^2]}\,\bar{F}_1, \tag{13a}$$

where

$$K_1 = k_1 - m_1^*\omega^2, \tag{13b}$$

$$K_2 = k_2 - m_2^* \omega^2. \tag{13c}$$

If the ratio of the spring mass, m_s, to the lumped mass of the absorber, m_2, is denoted by β, i.e.,

$$\beta = \frac{m_s}{m_2} \tag{14}$$

then, the substitution of Eq. (14) into Eq. (12) yields

$$m_s = \left(\frac{3\beta}{3+\beta}\right) m_2^*. \tag{15}$$

Introducing Eq. (15) into Eq. (13a) leads to

$$\bar{x}_1 = \frac{K_2 + j\omega c_2}{\bar{C} + j\bar{D}} \bar{F}_1, \tag{16a}$$

where

$$\bar{C} = K_1 K_2 - \left(1 + \frac{\beta}{3+\beta}\right) m_2^* k_2 \omega^2 - \frac{\beta^2}{4(3+\beta)^2} m_2^{*^2} \omega^4, \tag{16b}$$

$$\bar{D} = \omega c_2 \left[k_1 - \left(1 + \frac{\beta}{3+\beta}\right) m_2^* \omega^2 - m_1^* \omega^2\right]. \tag{16c}$$

From Eq. (16), one sees that $\bar{x}_1$ is a complex number and its magnitude is given by

$$|\bar{x}_1| = \bar{F}_1 \left[\frac{(\bar{C}K_2 + \bar{D}\omega c_2)^2 + (\bar{C}\omega c_2 - \bar{D}K_2)^2}{(\bar{C}^2 + \bar{D}^2)^2}\right]^{1/2} = \bar{F}_1 \left[\frac{K_2^2 + (\omega c_2)^2}{\bar{C}^2 + \bar{D}^2}\right]^{1/2}. \tag{17}$$

Therefore, the dynamic magnification factor for the main system is

$$\left|\frac{\bar{x}_1}{x_{\text{st}}}\right| = \left[\frac{K_2^2 + (\omega c_2)^2}{(\bar{C}^2 + \bar{D}^2)/k_1^2}\right]^{1/2}, \tag{18a}$$

where

$$x_{\text{st}} = \bar{F}_1 / k_1 \tag{18b}$$

is the displacement of lumped mass m_1 with respect to its static equilibrium position due to the external force amplitude $\bar{F}_1$.

Multiplying the numerator and denominator inside the square root of Eq. (18a) with $(m_1^*/m_2^* k_1)^2$, respectively, one obtains

$$\left|\frac{\bar{x}_1}{x_{\text{st}}}\right| = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} = \sqrt{\frac{P}{Q}}, \tag{19}$$

where

$$A = f^2 - \Omega^2, \tag{20}$$

$$B = 2\xi f \Omega, \tag{21}$$

$$C = (1 - \Omega^2)(f^2 - \Omega^2) - \left(1 + \frac{\beta}{3+\beta}\right) \mu f^2 \Omega^2 - \frac{\beta^2}{4(3+\beta)^2} \mu \Omega^4, \tag{22}$$

$$D = (2\xi f \Omega)\left[1 - \left(1 + \frac{\beta}{3+\beta}\right) \mu \Omega^2 - \Omega^2\right] \tag{23}$$

and

$$P = A^2 + B^2 = (f^2 - \Omega^2)^2 + (2\xi f\Omega)^2, \tag{24}$$

$$Q = C^2 + D^2 = \left[(1-\Omega^2)(f^2-\Omega^2) - \left(1+\frac{\beta}{3+\beta}\right)\mu f^2\Omega^2 - \frac{\beta^2}{4(3+\beta)^2}\mu\Omega^4\right]^2 + (2\xi f\Omega)^2\left[1-\left(1+\frac{\beta}{3+\beta}\right)\mu\Omega^2 - \Omega^2\right]^2. \tag{25}$$

In the last equations, ξ is the damping ratio, f and Ω are the frequency ratios and μ is the lumped mass ratio, they are, respectively, defined by

$$\xi = \frac{c_2}{2\sqrt{m_2^* k_2}}, \tag{26}$$

$$f = \frac{\sqrt{k_2/m_2^*}}{\sqrt{k_1/m_1^*}}, \tag{27}$$

$$\Omega = \frac{\omega}{\sqrt{k_1/m_1^*}} \tag{28}$$

$$\mu = \frac{m_2^*}{m_1^*}. \tag{29}$$

3.2. Optimum frequency ratio f_{opt}

Eq. (19) represents dynamic magnification factor of the main system. For the case of $\mu = 1/20$, $\beta = 0.1$, $f = 1.0$ and $\xi = 0.0, 0.1, 0.3$ and ∞, respectively, the curves for the dynamic magnification factor ($|\bar{x}_1/x_{st}|$) versus the frequency ratio $\left(\Omega = \omega/\sqrt{k_1/m_1^*}\right)$ are shown in Fig. 2. From the figure, one sees that all the curves pass through points R and S. In other words, at points R and S, the dynamic magnification factor is independent of damping ratio ξ. For this reason, one has

$$\left|\frac{\bar{x}_1}{x_{st}}\right|_{\xi=0} = \left|\frac{\bar{x}_1}{x_{st}}\right|_{\xi=\infty}. \tag{30}$$

From Eq. (13a), one finds that the responses at $\xi = 0$ (i.e., $c_2 = 0$) and $\xi = \infty$ (i.e., $c_2 = \infty$) are in opposite phase. Therefore, referring to Eqs. (13a)–(13c) and (19), one has

$$\frac{f^2-\Omega^2}{(1-\Omega^2)(f^2-\Omega^2) - \left(1+\frac{\beta}{3+\beta}\right)\mu f^2\Omega^2 - \frac{\beta^2}{4(3+\beta)^2}\mu\Omega^4} = \frac{-1}{1-\left(1+\frac{\beta}{3+\beta}\right)\mu\Omega^2 - \Omega^2} \tag{31}$$

or

$$\left[2 + \frac{4(3+2\beta)(3+\beta)-\beta^2}{4(3+\beta)^2}\mu\right]\Omega^4 - 2\left[\left(1+\frac{\beta}{3+\beta}\right)\mu f^2 + f^2 + 1\right]\Omega^2 + 2f^2 = 0. \tag{32}$$

Noted that $|\bar{x}_1/x_{st}| \approx A/C$ if $\xi = 0$ (or $c_2 = 0$) and $|\bar{x}_1/x_{st}| \approx -B/D$ if $\xi = \infty$ (or $c_2 = \infty$) as one may see from Eqs. (13), (16) and (19)–(23).

If the roots of the last equation are denoted by Ω_R and Ω_s, then from Eq. (32) one has

$$\Omega_R^2 + \Omega_S^2 = \frac{2\left[(1+\beta/(3+\beta))\mu f^2 + f^2 + 1\right]}{2 + \frac{4(3+2\beta)(3+\beta)-\beta^2}{4(3+\beta)^2}\mu}. \tag{33}$$

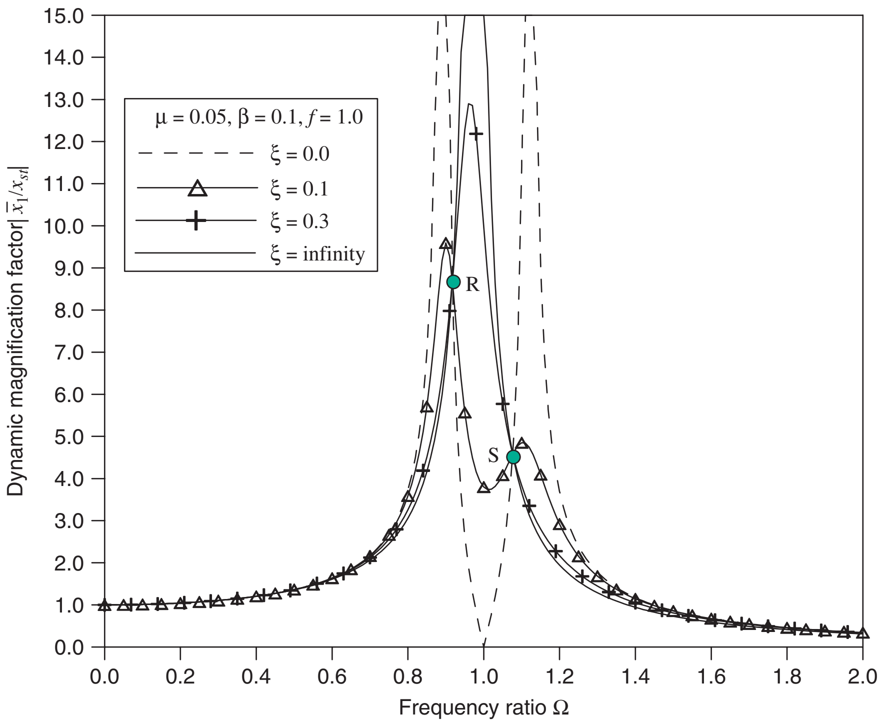

Fig. 2. Dynamic magnification factor $|\bar{x}_1/x_{st}|$ of the main system m_1 (see Fig. 1) carrying a modified absorber with lumped mass ratio $\mu = 0.05$, spring mass ratio $\beta = 0.1$, frequency ratio $f = 1.0$ and damping ratios $\xi = 0$, 0.1, 0.3 and ∞.

According to Ref. [3], the optimum parameters of the absorber can be achieved by adjusting the responses at points R and S to be equal and making R and S being the maximum points on the response curves. Based on this concept and Eqs. (30) and (31), one obtains

$$\frac{1}{(1+\beta/(3+\beta))\mu\Omega_R^2 - \Omega_R^2} = \frac{-1}{(1+\beta/(3+\beta))\mu\Omega_S^2 - \Omega_S^2} \tag{34}$$

or

$$\Omega_R^2 + \Omega_S^2 = \frac{2(3+\beta)}{(3+\beta)+(3+2\beta)\mu}. \tag{35}$$

From Eqs. (33) and (35), one obtains the optimum frequency ratio for the modified absorber

$$f_{\text{opt}} = \sqrt{\frac{4(3+\beta)^2 - \beta^2\mu}{4[(3+\beta)+(3+2\beta)\mu]^2}}. \tag{36}$$

3.3. Optimum damping ratio ξ_{opt}

Substituting Eq. (36) into Eq. (32) and solving the resulting expression for Ω_R and Ω_s will yield the frequency ratios corresponding to points R and S:

$$\Omega_R = \sqrt{(3+\beta)\frac{7\mu\beta^2 + 36\mu\beta - 6\mu\beta E_3 - 9\mu E_3 + 36\mu + 8\beta^2 + 48\beta - 3\beta E_3 - 9E_3 + 72}{E_1 E_2}}, \tag{37}$$

$$\Omega_S = \sqrt{(3+\beta)\frac{7\mu\beta^2 + 36\mu\beta + 6\mu\beta E_3 + 9\mu E_3 + 36\mu + 8\beta^2 + 48\beta + 3\beta E_3 + 9E_3 + 72}{E_1 E_2}}, \quad (38)$$

where

$$E_1 = 72 + 48\beta + 8\beta^2 + 36\mu + 36\mu\beta + 7\mu\beta^2, \quad (39)$$

$$E_2 = 3 + \beta + 3\mu + 2\mu\beta, \quad (40)$$

$$E_3 = \sqrt{\mu E_1(\beta+2)^2/E_2^2}. \quad (41)$$

If R and S are the maximum points on the response curve, then the slope for the tangent, at points R and S, of the last curve must be equal to zero, i.e.,

$$\frac{\partial}{\partial\Omega^2}\left[\frac{\bar{x}_1}{x_{\mathrm{st}}}\right]^2 = 0. \quad (42)$$

The substitution of Eq. (19) into Eq. (42) yields

$$P'Q - PQ' = 0, \quad (43)$$

where

$$P' = \frac{\partial P}{\partial\Omega^2} = -2(f^2 - \Omega^2) + 4\xi^2 f^2, \quad (44)$$

$$\begin{aligned} Q' = \frac{\partial Q}{\partial\Omega^2} = {} & 2[(1-\Omega^2)(f^2-\Omega^2) - \left(1+\frac{\beta}{3+\beta}\right)\mu f^2\Omega^2 - \frac{\beta^2}{4(3+\beta)^2}\,\mu\Omega^4]\cdot \\ & [-f^2 + 2\Omega^2 - 1 - \left(1+\frac{\beta}{3+\beta}\right)\mu f^2 - \frac{\beta^2}{2(3+\beta)^2}\,\mu\Omega^2] \\ & + 4\xi^2 f^2\left[1 - \left(1+\frac{\beta}{3+\beta}\right)\mu\Omega^2 - \Omega^2\right]^2 \\ & + 8\xi^2 f^2\Omega^2\left[1 - \left(1+\frac{\beta}{3+\beta}\right)\mu\Omega^2 - \Omega^2\right]\left[-\left(1+\frac{\beta}{3+\beta}\right)\mu - 1\right]. \end{aligned} \quad (45)$$

From Eqs. (19), (24), (25), (30) and (31), one sees that

$$\frac{P}{Q} = \frac{1}{[1-(1+\beta/(3+\beta))\mu\Omega^2 - \Omega^2]^2}. \quad (46)$$

Introducing Eq. (46) into Eq. (43) leads to

$$P'[1 - \left(1+\frac{\beta}{3+\beta}\right)\mu\Omega^2 - \Omega^2]^2 - Q' = 0. \quad (47)$$

Substituting Eqs. (44) and (45) into Eq. (47) and solving the resulting expression for ξ^2, one has

$$\xi^2 = \frac{J_\xi}{4f^2\Omega^2[1-(1+\beta/(3+\beta))\mu\Omega^2 - \Omega^2][-(1+\beta/(3+\beta))\mu - 1]}, \quad (48a)$$

where

$$J_{\xi} = -(f^2 - \Omega^2)\left[1 - \left(1 + \frac{\beta}{3+\beta}\right)\mu\Omega^2 - \Omega^2\right]^2 - \left[(1-\Omega^2)(f^2-\Omega^2) - \left(1+\frac{\beta}{3+\beta}\right)\mu f^2\Omega^2 - \frac{\beta^2}{4(3+\beta)^2}\,\mu\Omega^4\right] \times \left[-f^2 + 2\Omega^2 - 1 - \left(1+\frac{\beta}{3+\beta}\right)\mu f^2 - \frac{\beta^2}{2(3+\beta)^2}\,\mu\Omega^2\right]. \tag{48b}$$

Substituting Eqs. (36) and (37) into Eq. (48), one obtains the optimum damping ratio at point R to be

$$\xi^2 = \xi_R^2. \tag{49}$$

Similarly, one may obtain the optimum damping ratio at point S, by introducing Eqs. (36) and (38) into Eq. (48), to be

$$\xi^2 = \xi_S^2. \tag{50}$$

For convenience, we take the average of ξ_R^2 and ξ_S^2 as the optimum damping ratio of the modified absorber, i.e.,

$$\xi_{\text{opt}} = \sqrt{\frac{\xi_R^2 + \xi_S^2}{2}}. \tag{51}$$

From the existing literatures [1–5], it has been found that the approximate damping ratio $\xi_{\text{opt}} = \sqrt{(\xi_R^2 + \xi_S^2)/2}$ given by Eq. (51) is acceptable for the design of optimum parameters of vibration absorber. For this reason, Eq. (51) is used as the optimum damping ratio in this paper.

From the foregoing equations one sees that, if the values of μ and β are given, then one may determine the values of f_{opt}, Ω_R and Ω_s from Eqs. (36)–(38). The substitution of the last values of μ, β, f_{opt}, Ω_R and Ω_s into Eq. (48) will determine the values of ${\xi_R}^2$ and ${\xi_S}^2$. Finally, the optimum damping ratio ξ_{opt} is obtained from Eq. (51).

4. Dynamic magnification factor and optimum parameters for the classical absorber

The optimum parameters presented in the last section are for the modified absorber (with inertia effect of its spring mass considered). To validate the presented theory, the optimum parameters for the classical absorber (with inertia effect of its spring mass neglected) are given in this section. For the main system attached by a classical absorber as shown in Fig. 1, its dynamic magnification factor is given by [3]

$$\left|\frac{\bar{x}_1}{x_{\text{st}}}\right| = \sqrt{\frac{\bar{P}}{\bar{Q}}}, \tag{52}$$

where

$$\bar{P} = (\bar{f}^2 - \bar{\Omega}^2)^2 + (2\bar{\xi}\bar{f}\bar{\Omega})^2, \tag{53}$$

$$\bar{Q} = [(1-\bar{\Omega}^2)(\bar{f}^2 - \bar{\Omega}^2) - \bar{\mu}\bar{f}^2\bar{\Omega}^2]^2 + (2\bar{\xi}\bar{f}\bar{\Omega})^2[1 - \bar{\mu}\bar{\Omega}^2 - \bar{\Omega}^2]^2 \tag{54}$$

with

$$\bar{\xi} = \frac{c_2}{2\sqrt{m_2 k_2}}, \tag{55}$$

$$\bar{f} = \frac{\sqrt{k_2/m_2}}{\sqrt{k_1/m_1}}, \tag{56}$$

$$\bar{\Omega} = \frac{\omega}{\sqrt{k_1/m_1}}, \tag{57}$$

$$\bar{\mu} = \frac{m_2}{m_1}. \tag{58}$$

The optimum frequency and damping ratio of the classical absorber are [3]

$$\bar{f}_{\mathrm{opt}} = \frac{1}{1+\bar{\mu}}, \tag{59}$$

$$\bar{\xi}_{\mathrm{opt}} = \sqrt{\frac{3\bar{\mu}}{8(1+\bar{\mu})}}. \tag{60}$$

It is noted that the lumped mass ratio $\bar{\mu}$ defined in this section and that μ defined in the last subsection are different. If the value of μ is given, then that of $\bar{\mu}$ is determined by

$$\bar{\mu} = \frac{m_2}{m_1} = \frac{3\mu}{3+\beta-\mu\beta}. \tag{61}$$

The last equation is obtained by substituting Eqs. (11), (12) and (14) into Eq. (29).

If the spring mass of the absorber, m_s, is neglected, then from Eqs. (14), (58) and (61), one obtains

$$\beta = m_s/m_2 = 0, \tag{62a}$$

$$\mu = \bar{\mu} = m_2/m_1. \tag{62b}$$

Use of the last two relationships and Eqs. (36), (48)–(51), (59) and (60), one obtains

$$f_{\mathrm{opt}} = \bar{f}_{\mathrm{opt}} = \frac{1}{1+\bar{\mu}}, \tag{63a}$$

$$\xi_{\mathrm{opt}} = \bar{\xi}_{\mathrm{opt}} = \sqrt{\frac{3\bar{\mu}}{8(1+\bar{\mu})}}. \tag{63b}$$

It is believed that Eqs. (62b), (63a) and (63b) may be one of the good evidences for the reliability of the presented theory regarding the modified absorber.

5. Optimum parameters for the classical and modified absorbers attached to a beam

The optimum (non-dimensional) parameters for the absorber determined in the last two sections are for the case of the absorber to be attached to the single dof spring–mass main system (see Fig. 1). If the absorber is attached to a multiple dof pinned–pinned beam as shown in Fig. 5, the optimum (non-dimensional) parameters for the absorber may be determined by the technique presented in this section. Based on the last optimum (non-dimensional) parameters for the absorber, the corresponding optimum dimensional parameters (i.e., spring constant k_2 and damping coefficient c_2) of the absorber can be obtained.

5.1. Optimum non-dimensional parameters for the absorber attached to a beam

The equation of motion of the bare pinned–pinned beam (without carrying absorber) takes the form

$$[M]_{n'\times n'}\{\ddot{q}(t)\}_{n'\times 1} + [K]_{n'\times n'}\{q(t)\}_{n'\times 1} = \{F(t)\}_{n'\times 1}, \tag{64}$$

where $[M]$ and $[K]$ are, respectively, the effective overall mass and stiffness matrices, $\{\ddot{q}(t)\}$, $\{\dot{q}(t)\}$ and $\{q(t)\}$ are, respectively, the acceleration, velocity and displacement vectors, $\{F(t)\}$ is the external force vector and n' is the effective total degree of freedom of the pinned–pinned beam.

Based on the theory of mode superposition method and the orthogonal property of the normal mode shapes of the pinned–pinned beam, the n' coupled simultaneous differential equations given by Eq. (64) may be reduced to the following uncoupled ones [12]:

$$\tilde{m}_r \ddot{\eta}_r + \tilde{k}_r \eta_r = \tilde{f}_r (r = 1 - n'), \tag{65}$$

where

$$\tilde{m}_r = \{\phi_r\}^{\mathrm{T}}[M]\{\phi_r\}, \tag{66a}$$

$$\tilde{k}_r = \{\phi_r\}^{\mathrm{T}}[K]\{\phi_r\}, \tag{66b}$$

$$\tilde{f}_r = \{\phi_r\}^{\mathrm{T}}\{F(t)\}. \tag{66c}$$

In the last three equations, $\tilde{m}_r$, $\tilde{k}_r$ and $\tilde{f}_r$ are, respectively, the generalized mass, generalized stiffness and generalized force for the *equivalent* single dof spring–mass system associated with the rth vibration mode of the pinned–pinned beam, $\ddot{\eta}_r$ and η_r are, respectively, the rth generalized acceleration and displacement, while $\{\phi_r\}$ is the rth normal mode shape.

For a pinned–pinned beam subjected to a moving load as shown in Fig. 5, the contribution to the dynamic responses of the beam from its first mode (i.e., $r = 1$) is the most significant. Therefore, one may design an optimum absorber according to the generalized mass and generalized stiffness of the *equivalent* single dof spring–mass system associated with the 1st vibration mode of the pinned–pinned beam, $\tilde{m}_1$ and $\tilde{k}_1$. It is evident that the values of $\tilde{m}_1$ and $\tilde{k}_1$ may be obtained from Eqs. (66a) and (66b) with $r = 1$. If the lumped mass m_2 and the spring mass m_s of the absorber are given, then the associate optimum non-dimensional parameters for the modified (or classical) absorber may be obtained from Eqs. (14), (29), (36) and (51) (or Eqs. (58)–(61)).

5.2. *Optimum dimensional parameters for the absorber attached to a beam*

If the lumped mass m_2 and the ratio of spring mass m_s to lumped mass m_2 of the modified absorber, $\beta = m_s/m_2$, are given, then the lumped mass ratio for the effective lumped mass of the modified absorber, m_2^*, to the effective generalized mass associated with the 1st vibration mode of the beam, m_1^*, is determined by (cf. Eq. (29))

$$\mu = \frac{m_2^*}{m_1^*} = \frac{m_2 + (m_s/3)}{\tilde{m}_1 + (m_s/3)} = \frac{3m_2 + \beta m_2}{3\tilde{m}_1 + \beta m_2}. \tag{67}$$

Substituting the values of β and μ into Eqs. (36) and (48)–(51) will determine the values of optimum frequency ratio f_{opt} and optimum damping ratio ξ_{opt}. Thus, based on Eqs. (27) and (26), the corresponding optimum spring constant and optimum damping coefficient of the modified absorber are, respectively, given by

$$k_2 = m_2^* \left(\frac{k_1}{m_1^*}\right) f_{\mathrm{opt}}^2 = (m_2 + \tfrac{1}{3}\beta m_2) \left[\frac{\tilde{k}_1}{\tilde{m}_1 + \tfrac{1}{3}\beta m_2}\right] f_{\mathrm{opt}}^2, \tag{68a}$$

$$c_2 = 2\xi_{\mathrm{opt}} \sqrt{m_2^* k_2} = 2\xi_{\mathrm{opt}} \sqrt{(m_2 + \tfrac{1}{3}\beta m_2) k_2}. \tag{68b}$$

Eqs. (68a) and (68b) are for determining the spring constant and damping coefficient of the *modified* absorber associated with 1st vibration mode of the beam. For the case of *classical* absorber, Eqs. (68a) and (68b) must be replaced by

$$k_2 = m_2 \left(\frac{k_1}{m_1}\right) \bar{f}_{\mathrm{opt}}^2 = m_2 \left(\frac{\tilde{k}_1}{\tilde{m}_1}\right) \bar{f}_{\mathrm{opt}}^2, \tag{69a}$$

$$c_2 = 2\bar{\xi}_{\mathrm{opt}} \sqrt{m_2 k_2}, \tag{69b}$$

where $\bar{f}_{\mathrm{opt}}$ and $\bar{\xi}_{\mathrm{opt}}$ can be obtained from Eqs. (58) to (60).

6. Dynamic responses of a beam due to a moving load

In this section, the theories for the determinations of dynamic responses of a uniform beam, without and with absorber, subjected to a moving concentrated load are presented.

6.1. Dynamic responses of a beam without absorber and subjected to a moving load

The equation of motion for a multiple dof beam without absorber (cf. Fig. 3) is given by

$$[\bar{M}]\{\ddot{\bar{q}}(t)\} + [\bar{C}]\{\dot{\bar{q}}(t)\} + [\bar{K}]\{\bar{q}(t)\} = \{\bar{F}(t)\}, \tag{70}$$

where $[\bar{M}]$, $[\bar{C}]$ and $[\bar{K}]$ are, respectively, the overall mass, damping and stiffness matrices, $\{\ddot{\bar{q}}(t)\}$, $\{\dot{\bar{q}}(t)\}$ and $\{q(t)\}$ are, respectively, the acceleration, velocity and displacement vectors of the entire *unconstrained* beam and $\{\bar{F}(t)\}$ is the associated overall external force vector.

In this paper, the overall stiffness and mass matrices, $[\bar{K}]$ and $[\bar{M}]$, of the entire beam are determined by assembling the elementary property matrices for each of the beam elements [12]. Since the beam studied is assumed to be undamped, its overall damping matrix is a zero matrix, i.e., $[\bar{C}] = [0]$.

When the beam is subjected to a concentrated force P_c, all the nodal forces of the beam are equal to zero except those for the sth beam element on which the concentrated force P_c applies (see Fig. 4) [13,14]. Thus, the overall external force vector $\{\bar{F}(t)\}$ in Eq. (70) takes the form

$$\{\bar{F}(t)\} = [000\cdots f_1^{(s)}(t) f_2^{(s)}(t) f_3^{(s)}(t) f_4^{(s)}(t)\cdots 000]^{\mathrm{T}}, \tag{71}$$

where $f_i^{(s)}(t)$ $(i = 1-4)$ are the equivalent nodal forces of the sth beam element on which the moving load P_c applies and are given by [14]

$$\{f^{(s)}(t)\} = [f_1^{(s)}(t) \quad f_2^{(s)}(t) \quad f_3^{(s)}(t) \quad f_4^{(s)}(t)]^{\mathrm{T}} = P_c\{N\} \tag{72}$$

with

$$\{N\} = [N_1 \quad N_2 \quad N_3 \quad N_4]^{\mathrm{T}}. \tag{73}$$

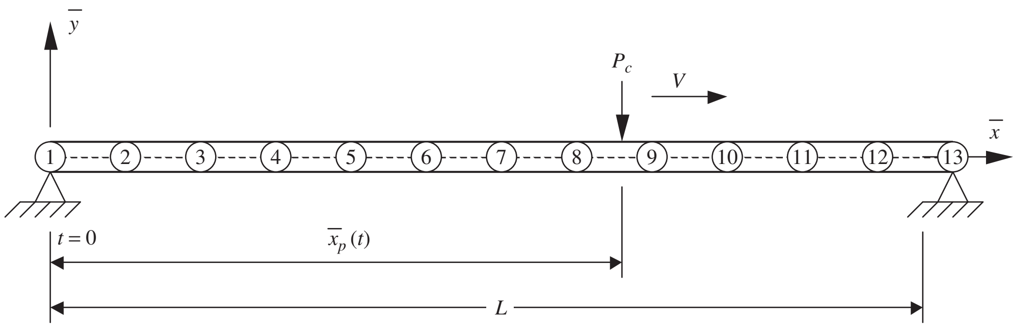

Fig. 3. A pinned–pinned beam subjected to a concentrated force P_c moving from the left-end to the right-end of the beam with a constant speed V.

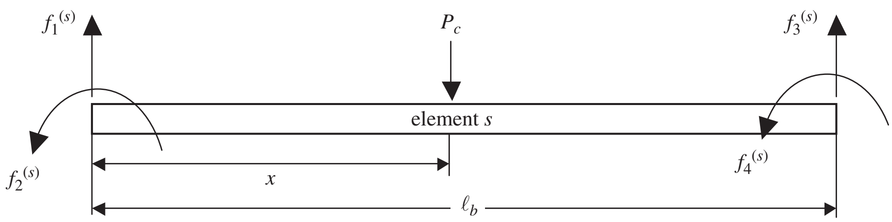

Fig. 4. Equivalent nodal forces $f_i^{(s)}$ $(i = 1-4)$ of the sth beam element due to a concentrated force P_c.

The symbols N_i ($i = 1-4$) appearing in Eq. (73) are the shape functions of the beam element given by [14]

$$N_1 = 1 - 3\varsigma^2 + 2\varsigma^3, \tag{74a}$$

$$N_2 = (\varsigma - 2\varsigma^2 + \varsigma^3)\ell_b, \tag{74b}$$

$$N_3 = 3\varsigma^2 - 2\varsigma^3, \tag{74c}$$

$$N_4 = (-\varsigma^2 + \varsigma^3)\ell_b \tag{74d}$$

with

$$\varsigma = x/\ell_b, \tag{74e}$$

where ℓ_b and x, respectively, represent the length of the beam element and the distance between the position of the concentrated force P_c and the left-end of the beam element (see Fig. 4).

It is noted that, in Eq. (71), the equivalent nodal forces, $f_i^{(s)}(t)$ ($i = 1-4$) are, respectively, the s_1th, s_2th, s_3th and s_4th coefficients of the overall external force vector $\{F(t)\}$, where s_1, s_2, s_3 and s_4 are, respectively, the numberings for the four degrees of freedom of the sth beam element on which the moving load P_c applies.

If the concentrated force P_c moves, with a constant velocity V, from the left-end to the right-end of the beam (see Fig. 3), the position of the concentrated force P_c at any instant of time t is given by

$$\bar{x}_p(t) = Vt. \tag{75}$$

Thus, the numbering for the beam element on which the concentrated force P_c applies at time t is determined by

$$s = \left(\text{Integer part of } \frac{\bar{x}_p(t)}{\ell_b}\right) + 1. \tag{76}$$

Now, one may use Eqs. (71)–(76) to calculate the instantaneous overall external force vector $\{F(t)\}$, appearing in Eq. (70), at any instant of time t. It is noted that the local x coordinate of the moving concentrated force P_c, as shown in Eq. (74e), is a function of the global coordinate $\bar{x}_p(t)$, i.e.,

$$\varsigma = \frac{x}{\ell_b} = \frac{\bar{x}_p(t) - (s-1)\ell_b}{\ell_b}. \tag{77}$$

Finally, one may calculate the dynamic responses of the *bare* beam (i.e., the beam without any attachment, such as the absorber) due to a moving force.

6.2. Dynamic responses of a beam with an absorber and subjected to a moving load

The formulations of the last section are for calculating the dynamic responses of the pinned–pinned beam *without* absorber (see Fig. 3) due to a moving load. For the forced vibration responses of the same beam *with* an absorber as shown in Fig. 5, the overall mass matrix $[\hat{M}]$, damping matrix $[\hat{C}]$ and stiffness matrix $[\hat{K}]$ of the entire vibrating system (i.e., the beam with an absorber) must be determined as follows:

$$[\hat{M}]_{(n+1)\times(n+1)} = [\bar{M}]_{n\times n} + [m]_{2\times 2}, \tag{78a}$$

$$[\hat{C}]_{(n+1)\times(n+1)} = [\bar{C}]_{n\times n} + [c]_{2\times 2}, \tag{78b}$$

$$[\hat{K}]_{(n+1)\times(n+1)} = [\bar{K}]_{n\times n} + [k]_{2\times 2}, \tag{78c}$$

where $[\bar{M}]$, $[\bar{C}]$ and $[\bar{K}]$ are, respectively, the overall mass, damping and stiffness matrices of the *unconstrained* beam itself with total dof n, and $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrices of the absorber. If the inertia effect of the spring mass of the absorber is considered, then the last element property matrices are given by Eqs. (7e), (7f) and (7g), respectively. If the inertia effect of the spring mass of the absorber is

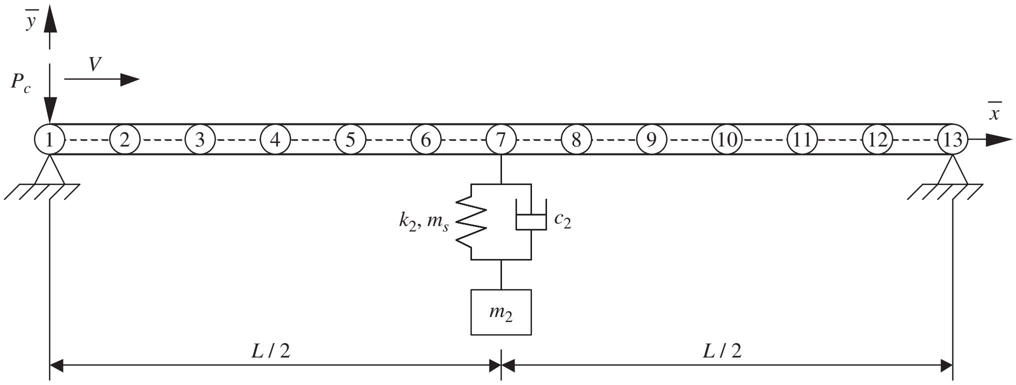

Fig. 5. A pinned–pinned beam with a central absorber and subjected to a concentrated force P_c moving from the left-end to the right-end of the beam with a constant speed V.

neglected, then the element property matrices of the absorber are exactly the same as those defined by Eqs. (7e)–(7g) except that the mass matrix must be replaced by [13,14]

$$[m] = \begin{bmatrix} 0 & 0 \\ 0 & m_2 \end{bmatrix}. \tag{79}$$

It is noted that, in Eqs. (78), the addition of $[m]$, $[c]$ and $[k]$ to $[\bar{M}]$, $[\bar{C}]$ and $[\bar{K}]$ must be conducted according to the numberings for the dof of the absorber. The overall property matrices, $[\hat{M}]$, $[\hat{C}]$ and $[\hat{K}]$, obtained from Eq. (78) are for the *unconstrained* beam. The effective overall property matrices of the pinned–pinned beam, $[M]$, $[C]$ and $[K]$, are obtained from the overall matrices $[\hat{M}]$, $[\hat{C}]$ and $[\hat{K}]$ by imposing the prescribed boundary conditions. It is evident that the order of the effective overall property matrices for the loaded beam (carrying n_{ab} absorbers) is greater than that of the bare beam by n_{ab}, where n_{ab} denotes the total number of absorbers carried by the beam.

7. Numerical results and discussions

In this section, numerical example is illustrated to validate the presented theory and the developed computer program first. Then, the vibration–reduction efficiency of an absorber on a uniform pinned–pinned beam subjected to a moving concentrated load is studied.

7.1. Validation

The example illustrated in this subsection is the single dof spring–mass main system carrying an (spring–damper–mass) absorber, as shown in Fig. 1. The curve for the dynamic magnification factor of the lumped mass m_1 of the main system, $|\bar{x}_1/x_{st}|$, versus frequency ratio, $\Omega = \omega\big/\sqrt{k_1/m_1^*}$, is obtained from Eq. (19) based on spring mass ratio $\beta = 0.0$ and lumped mass ratio $\mu = 0.05$ together with the optimum parameters for the *modified absorber* ($f = 0.9524$, $\xi = 0.1336$), and then it is compared with the one obtained from Eq. (52) by using the lumped mass ratio $\bar{\mu} = 0.05$ together with the optimum parameters for the *classical absorber* ($\bar{f} = 0.9524$, $\bar{\xi} = 0.1336$). The result is shown in Fig. 6, in which, the solid curve with circles (—○—) is for the modified absorber and the solid curve with crosses (— × —) is for the classical absorber. Good agreement between the last two curves confirms the reliability of the presented theory.

It is noted that the last two sets of optimum parameters, i.e., ($f = 0.9524$, $\xi = 0.1336$) for the modified absorber and ($\bar{f} = 0.9524$, $\bar{\xi} = 0.1336$) for the classical absorber, are obtained from Eqs. (36) and (51) and

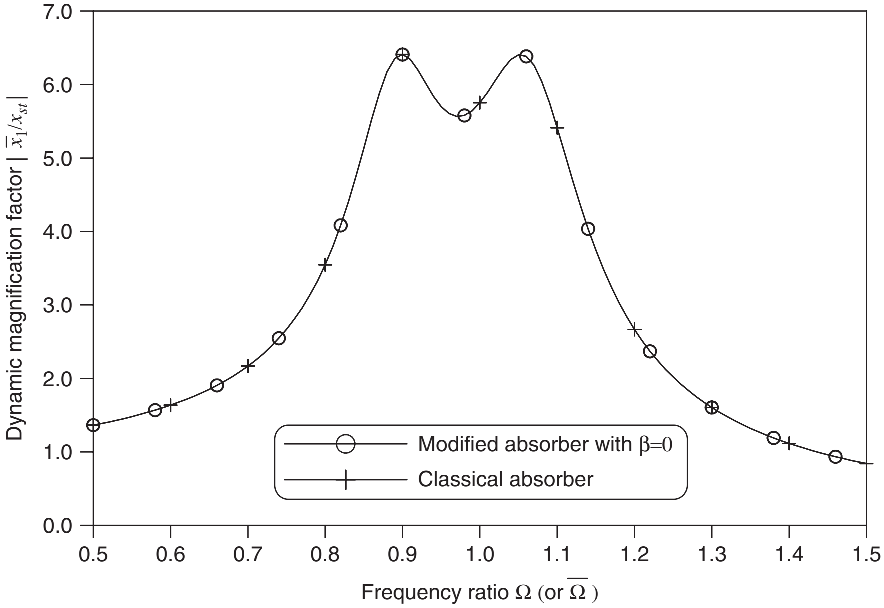

Fig. 6. Dynamic magnification factor $|\bar{x}_1/x_{\rm st}|$ of the main system m_1 (see Fig. 1) attached by the modified absorber with optimum parameters ($f = 0.9524$, $\xi = 0.1336$) and that attached by the classical absorber with optimum ones ($\bar{f} = 0.9524$, $\bar{\xi} = 0.1336$) based on zero spring mass ratio (i.e., $\beta = 0.0$) and constant lumped mass ratio $\bar{\mu} = \mu = 0.05$.

(58)–(60), respectively, based on the given data: $\beta = 0.0$ and $\mu = \bar{\mu} = 0.05$. They are exactly the same, because the only difference between the modified absorber and the classical absorber is that the former considers its spring mass and this is not true for the latter, and the mass ratio for the current modified absorber is assumed to be zero, i.e., $\beta = 0$.

7.2. *Influence of mass ratio β*

The curves of dynamic magnification factor $|\bar{x}_1/x_{\rm st}|$ versus frequency ratio Ω for the main system m_1 (see Fig. 1) are dependent on the spring mass ratio of the modified absorber, $\beta = m_s/m_2$. The influence of spring mass ratio is investigated here, with $\beta = 0.025$, 0.05, 0.075 and 0.1, respectively. The data required for plotting the last curves are shown in Table 1. In which, the optimum parameters for the modified absorber, $\xi_{\rm opt}$ and $f_{\rm opt}$, are determined from Eqs. (51) and (36), while those for the classical absorber, $\bar{\xi}_{\rm opt}$ and $\bar{f}_{\rm opt}$, are from Eqs. (60) and (59). It is noted that the lumped mass ratios $\bar{\mu}$ of the classical absorber listed in the third column of Table 1 are determined from the formula $\bar{\mu} = 3\mu/(3+\beta-\mu\beta)$ given by Eq. (61) using the corresponding data for the modified absorber with its values of β and μ listed in first and second columns of Table 1.

Figs. 7(a)–(d) show the curves of dynamic magnification factor $|\bar{x}_1/x_{\rm st}|$ versus frequency ratio Ω for the main system m_1 for the cases of spring mass ratios $\beta = 0.025$, 0.05, 0.075 and 0.1, respectively. In which, the solid curves with circles (—○—) are for the modified absorber and those with crosses (— × —) are for the classical absorber. From the figure, one sees that the differences between the last two curves increase with increasing the mass ratio β. Besides, the maximum dynamic magnification factor of the optimum modified absorber is always smaller than that of the optimum classical absorber. In general, the helical spring of the absorber is not massless. Therefore, the presented optimum modified absorber should be more reasonable than the optimum classical one.

7.3. *Vibration suppression of a beam with an absorber and subjected to a moving load*

To show the applicability of the presented theory, the optimum parameters of an absorber for suppressing the forced vibration responses of a pinned–pinned beam subjected to a concentrated force $P_c = 9.8\ N$ moving

Table 1
Influence of spring mass ratio ($\beta = m_s/m_2$) of the absorber on the optimum parameters of the modified absorber (with $\beta = m_s/m_2$) and classical absorber (with $\beta = 0$) based on constant lumped mass ratio $\mu = m_2^*/m_1^* = 0.05$ (see Fig. 1)

Spring mass ratios of absorber $\beta = m_s/m_2$	Lumped mass ratios		Optimum parameters			
	Modified absorber $\mu = m_2^*/m_1^*$	Classical absorber $\bar{\mu} = m_2/m_1$	Modified absorber ($\beta = m_s/m_2$)		Classical absorber ($\beta = 0$)	
			ξ_{opt}	f_{opt}	$\bar{\xi}_{opt}$	$\bar{f}_{opt}$
0.025	0.05	0.0496	0.1342	0.9520	0.1331	0.9527
0.05		0.0492	0.1347	0.9516	0.1326	0.9531
0.075		0.0488	0.1352	0.9513	0.1321	0.9534
0.1		0.0485	0.1357	0.9509	0.1317	0.9538

Note: $\bar{\mu} = 3\mu/(3 + \beta - \mu\beta)$.

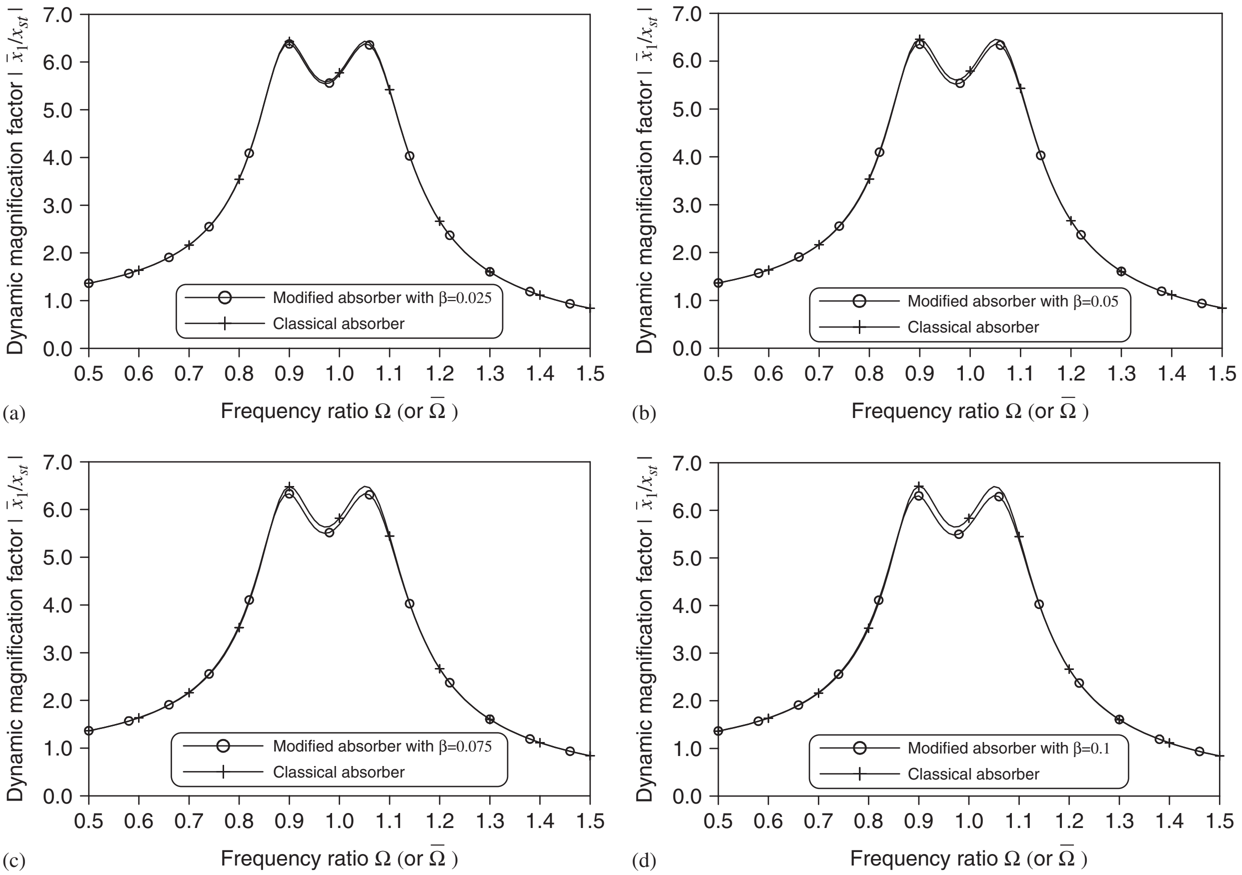

Fig. 7. Influence of spring mass ratio ($\beta = m_s/m_2$) of the absorber on the dynamic magnification factor $|\bar{x}_1/x_{st}|$ of the main system m_1 (see Fig. 1), respectively, attached by a modified absorber (with constant lumped mass ratio $\mu = m_2^*/m_1^* = 0.05$) and a classical absorber: (a) $\beta = 0.025$, (b) $\beta = 0.05$, (c) $\beta = 0.075$, (d) $\beta = 0.1$.

from the left-end to the right-end of the beam (see Figs. 3 and 5) are investigated. The beam, composed of 13 nodes and 12 identical beam elements, is made of steel with mass density $\rho = 7820\,\mathrm{kg/m^3}$, Young's modulus $E = 206.8\,\mathrm{GN/m^2}$, total length $L = 4\,\mathrm{m}$ and cross sectional area $A = 0.03\,\mathrm{m} \times 0.03\,\mathrm{m}$.

For a pinned–pinned beam subjected to a moving load, the contribution to the dynamic responses of the beam from its first mode is the most significant. Therefore, this section will design an optimum absorber according to the modal data of the first vibration mode of the beam. To this end, an absorber will be installed to the central point of the beam (i.e., the crest of the first mode shape of the beam), as shown in Fig. 5. According to Eqs. (66a) and (66b), the generalized mass $\tilde{m}_1$ and generalized stiffness $\tilde{k}_1$ associated with the 1st vibration mode of the beam are, respectively, given by $\tilde{m}_1 = 14.0764\,\text{kg}$ and $\tilde{k}_1 = 10621.3\,\text{N/m}$. Theoretically, the larger the mass of the absorber, the better the vibration–reduction efficiency of the absorber. However, the static deflection of the beam also increases with increasing the lumped mass of the absorber. Therefore, the lumped mass of the absorber cannot be too large. In this subsection, the lumped mass of the absorber is taken to be 5% of total mass of the pinned–pinned beam, i.e., $m_2 = 7820 \times 0.03 \times 0.03 \times 4 \times 0.05 = 1.4076\,\text{kg}$.

Based on the above-mentioned generalized mass $\tilde{m}_1 = 14.0764\,\text{kg}$ and generalized stiffness $\tilde{k}_1 = 10621.3\,\text{N/m}$ together with the constant lumped mass of the absorber $m_2 = 1.4076\,\text{kg}$ and Eqs. (14), (29), (36), (51) and (58)–(61), one obtains the spring mass ratios μ (and $\bar{\mu}$), optimum frequency ratios f_{opt} ($\bar{f}_{\text{opt}}$), optimum damping ratios ξ_{opt} (and $\bar{\xi}_{\text{opt}}$) of the modified (and classical) absorbers for the cases of $\beta = 0, 0.1, 0.2$ 0.3 and 0.4, respectively, as shown in Table 2. Next, based on the last information, the corresponding optimum spring constants k_2 and optimum damping coefficients c_2 of the modified and classical absorbers are also calculated according to Eqs. (68) and (69) and shown in Table 3.

Table 2
Optimum non-dimensional parameters for the modified absorber (with $\beta = m_s/m_2$) and classical absorber (with $\beta = 0$) with constant lumped mass of absorber, $m_2 = 1.4076\,\text{kg}$, for suppressing the vibration responses of a beam subjected to a moving load, for the cases of $\beta = m_s/m_2 = 0.0$, 0.1, 0.2, 0.3 and 0.4 (see Fig. 5)

Spring mass ratios of absorber $\beta = m_s/m_2$	Lumped mass ratios		Optimum non-dimensional parameters			
	Modified absorber $\mu = m_2^*/m_1^*$	Classical absorber $\bar{\mu} = m_2/m_1$	Modified absorber ($\beta = m_s/m_2$)		Classical absorber ($\beta = 0$)	
			ξ_{opt}	f_{opt}	$\bar{\xi}_{\text{opt}}$	$\bar{f}_{\text{opt}}$
0.0	0.1	0.1	0.184637	0.909090	0.184637	0.90909
0.1	0.102987		0.189853	0.903894		
0.2	0.105957		0.194885	0.898766		
0.3	0.108908		0.199744	0.893707		
0.4	0.111839		0.204437	0.888720		

Table 3
The corresponding optimum dimensional parameters for the modified absorber (with $\beta = m_s/m_2$) and classical absorber (with $\beta = 0$) with constant lumped mass of absorber, $m_2 = 1.4076\,\text{kg}$, and with their optimum non-dimensional parameters shown in Table 2 (see Fig. 5)

Spring mass ratios of absorber $\beta = m_s/m_2$	Spring mass of absorber m_s (kg)	Lumped masses		Optimum dimensional parameters			
		Modified absorber m_2 (kg)	Classical absorber m_2 (kg)	Modified absorber ($\beta = m_s/m_2$)		Classical absorber ($\beta = 0$)	
				c_2 (N s/m)	k_2 (N/m)	c_2 (N s/m)	k_2 (N/m)
0.0	0.0			12.980	877.767	12.980	877.767
0.1	0.14076	1.4076		13.690	893.708		
0.2	0.28152			14.400	909.080		
0.3	0.42228			15.120	923.905		
0.4	0.56304			15.818	938.212		

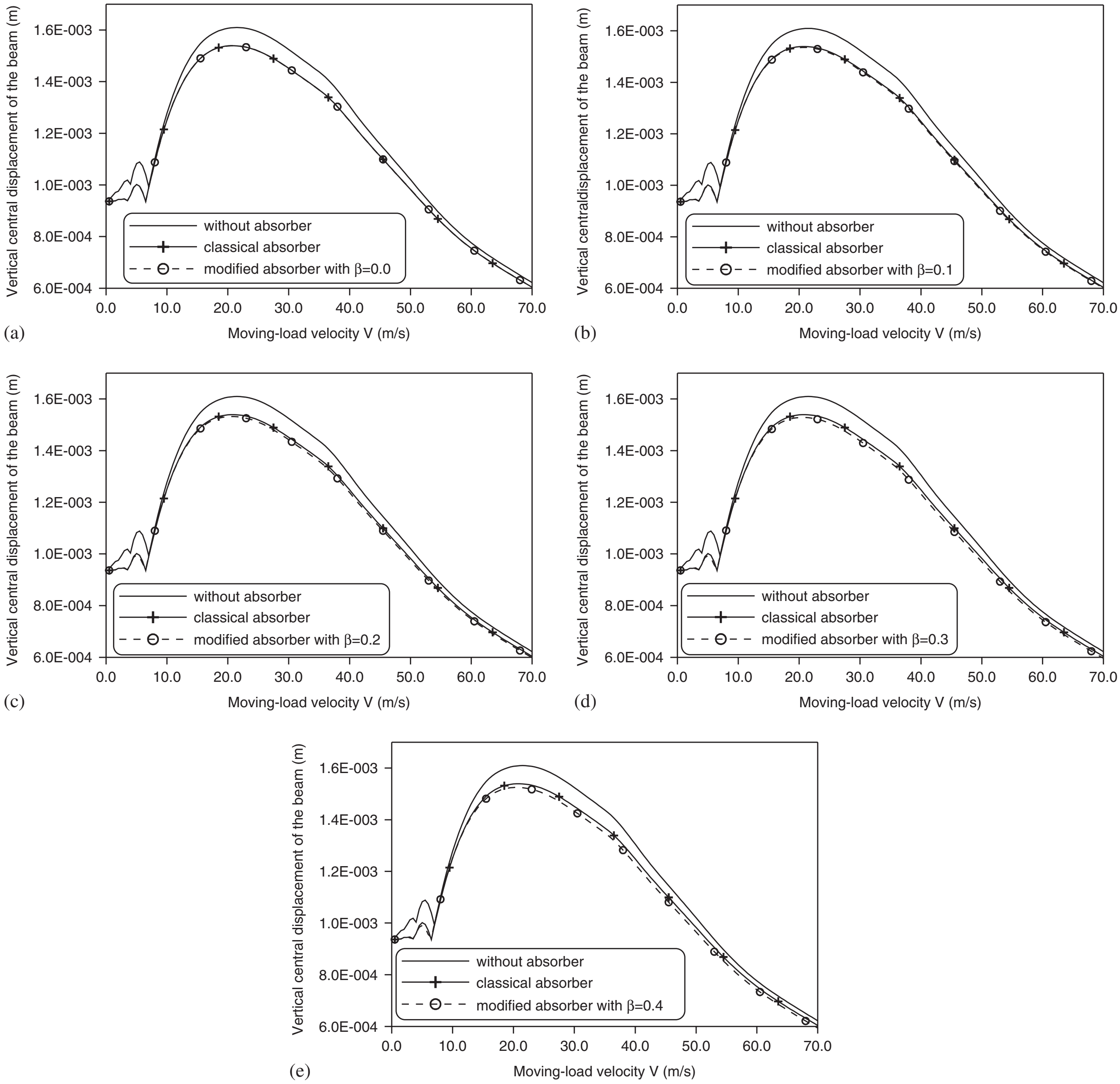

Fig. 8. Influence of spring mass ratio ($\beta = m_s/m_2$) of the absorber and moving-load speed V on the maximum dynamic responses of the pinned–pinned beam, respectively, attached by a central modified absorber and a classical absorber with constant lumped mass of absorber $m_2 = 1.4076\,\mathrm{kg}$ and subjected to a moving load with magnitude $P_c = 9.8\,\mathrm{N}$ (see Fig. 5): (a) $\beta = 0$, (b) $\beta = 0.1$, (c) $\beta = 0.2$, (d) $\beta = 0.3$, (e) $\beta = 0.4$.

By means of the data given by Tables 2 and 3, one obtains the maximum vertical ($\bar{y}$) displacements of the central point of the pinned–pinned beam subjected to a moving load, with magnitude $P_c = 9.8\,\mathrm{N}$ and constant speed $V = 0-70\,\mathrm{m/s}$, as shown in Figs. 8(a)–(e) for the cases of $\beta = 0$, 0.1, 0.2, 0.3 and 0.4, respectively. From these figures, one finds that the larger the spring mass ratio β, the smaller the maximum vertical central displacements of the pinned–pinned beam.

From the foregoing discussions, one sees that the mass of the helical spring of the absorber does affect the vibration–reduction efficiency of the absorber to some degree. Therefore, it is significant to take the spring mass of the absorber into the formulations.

8. Conclusions

Because of complexity of the mathematical expressions, the mass of the helical spring (or spring mass) is usually neglected in the dynamic analysis of the *classical absorber*. In order to realize the influence of the spring mass, this paper has compared the characteristics of the last classical absorber with those of the *modified absorber* by taking the inertia effect of its spring mass into consideration. The examples studied include the dynamic responses of a single dof spring–mass main system, respectively, attached by a modified absorber and a classical absorber and subjected to a harmonic excitation, and the dynamic responses of a multiple dof pinned–pinned beam, respectively, carrying a modified absorber and a classical absorber and subjected to a moving concentrated load. The absorbers adopted are the types of spring–damper–mass systems. The curves for the dynamic responses of the main system (the single dof spring–mass system or the multiple dof pinned–pinned beam) reveal that the vibration–reduction efficiency of the absorber is dependent on the inertia effect of its spring mass to some degree. In view of the fact that the helical spring of the absorber is not massless, the presented modified absorber (with spring mass considered) should be more practical than the classical absorber (with spring mass neglected).

From the element mass matrix of the modified absorber given by Eq. (7e), one sees that the spring mass of the modified absorber affects both the effective mass of the main system and that of the modified absorber, besides, it has also the effect of coupling the above two systems. However, this is not true for the lumped mass of the classical absorber. The above-mentioned different effect is due to the fact that the spring mass of the modified absorber is the *distributed* mass, but the lumped mass of the classical absorber is the *concentrated* mass.

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